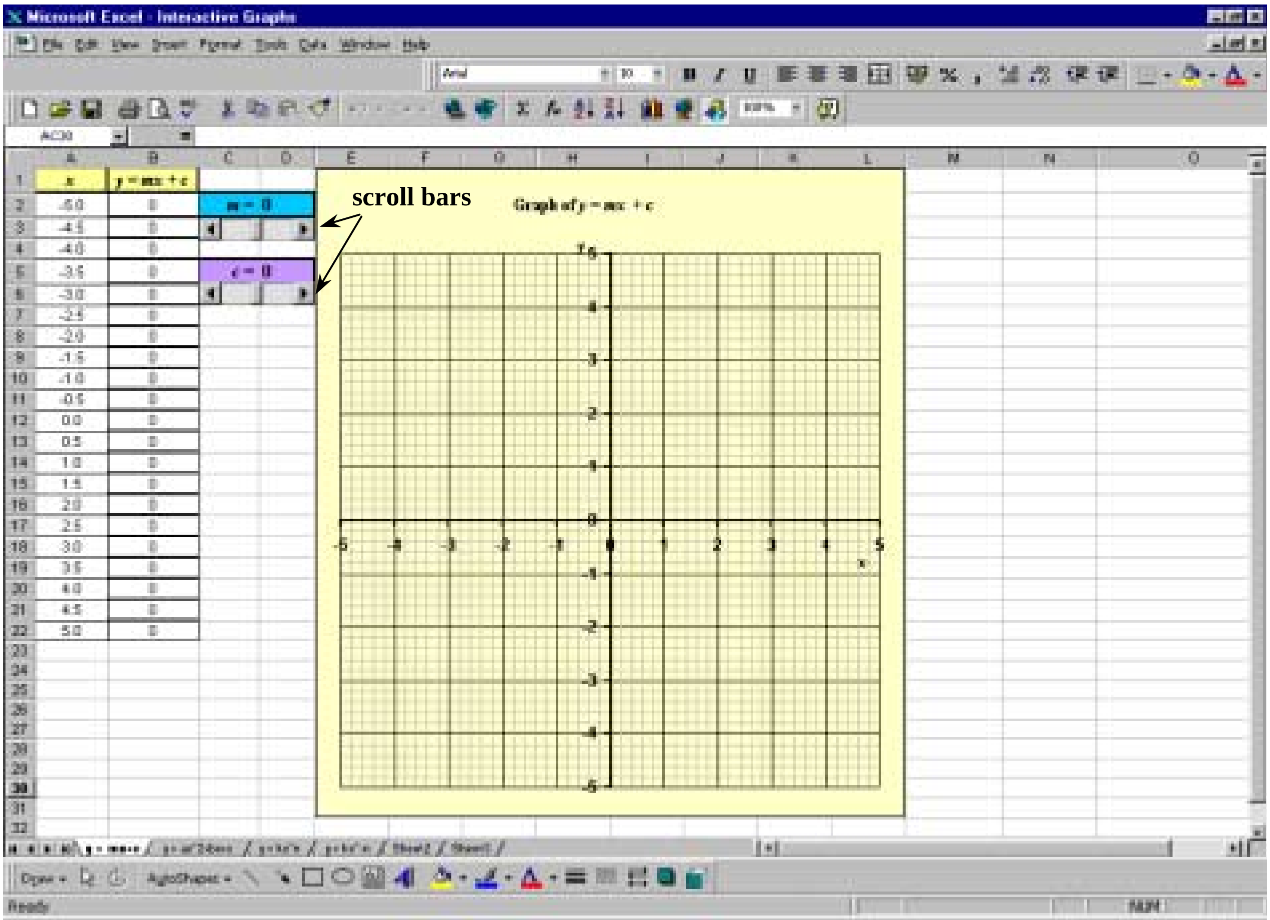
Interactive Graphs

Open Interactive Graphs.xls
The first sheet is shown below. It is set up to show graphs with equations of the form $y = mx + c$
At present the values of m and c are both zero. You can change these values using the scroll bars.

Leave the value of c equal to zero, and use the scroll bar to change the value of m.
Try using different parts of the scroll bar to see what happens and look at the tables and graphs you get.



You have been looking at some graphs with equations of the form $y = mx$ (since c is zero).
In each case y is ***proportional*** to x.

Answer these questions about graphs with equations of this type (i.e. $y = mx$)

1 What point on the graph does the line ***always*** pass through?…………………………………………….

2 What happens to the line as the value of m increases from 0 to 5?

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3 What happens to the line as the value of m decreases from 0 to – 5?

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Now keep m = 0 and use the scroll bar to change the value of c.
Answer these questions about such graphs i.e. graphs with equations of the form $y = c$

4 What happens to the line as the value of *c* increases from – 4 to 4?

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Answer these questions about graphs with equations $y = mx + c$
Keep m = 1 and use the scroll bar to change the value of c.

5 What happens to the line as the value of *c* increases from – 4 to 4?

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Keep m = – 1 and change the value of c.

6 What happens to the line as the value of *c* increases from – 4 to 4?

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Try other values of m and c.

7 What can you say about the link between the line and the value of *c*?

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8 What can you say about the link between the line and the value of *m*?

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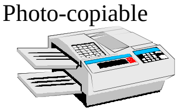
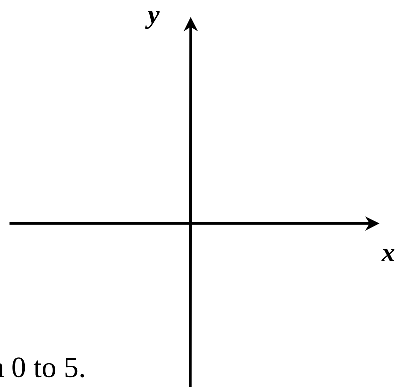


Quadratic Graphs

Use the second worksheet (y = ax^2 + bx + c) of Interactive Graphs.xls
It is set up to show graphs with equations of the form $y = ax^2 + bx + c$

Questions 1 to 4 are about curves with equations of the form $y = ax^2$
Keep b and c at zero and experiment with different values of a.



1 On this set of axes sketch the graph when a is positive.

2 Describe what happens to the curve as the value of a increases from 0 to 5.

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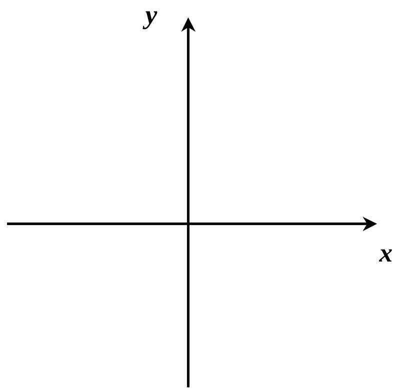

3 On this set of axes sketch the graph when a is negative.

4 Describe what happens to the curve as the value of a decreases from 0 to – 5.

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Questions 5 to 7 are about curves with equations of the form $y = ax^2 + c$
Keep a = 1 and b = 0. Use the scroll bar to change the value of c.
5 What happens to the curve as the value of c increases from – 4 to 4?

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Keep a = – 1 and b = 0. Change the value of c.
6 What happens to the curve as the value of c increases from – 4 to 4?

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Try other values of a and c, keeping b = 0.
7 What can you say about the link between the curve and the values of a and c?

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Questions 8 to 11 are about curves with equations of the form $y = ax^2 + bx$

Keep a = 1 and c = 0 and change the value of b.

8 Describe what happens to the curve as the value of *b* increases from 0 to 4.
Write down anything you notice.

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9 Describe what happens to the curve as the value of *b* decreases from 0 to – 4.

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Now keep a = – 1 and c = 0 and change the value of b.

10 Describe what happens to the curve as the value of *b* increases from 0 to 4?
Write down anything you notice.

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11 Describe what happens to the curve as the value of *b* decreases from 0 to – 4?

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Experiment with different values of a, b and c.

Check what effect changing each constant has on curves with equations of the form $y = ax^2 + bx + c$ **.**

Write a summary of your findings below.

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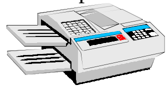

Power Graphs

Use the third worksheet (y = kx^n) of Interactive Graphs.xls

It is set up to show graphs with equations of the form $y = kx^n$ where n is a positive power.

1 ***Keeping k = 1 click on the scroll bar to the right of the central divider to increase the value of n from 0 to 1, then 2, 3, 4, ….10.***

a Explain why the graph is a straight line when n = 0 and n = 1.

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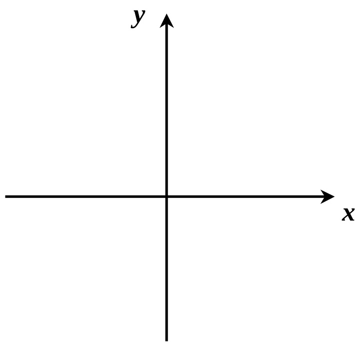
b On this set of axes sketch the general shape of the graph $y = kx^n$ when n is a positive even integer.
Describe what happens to the curve as n increases.



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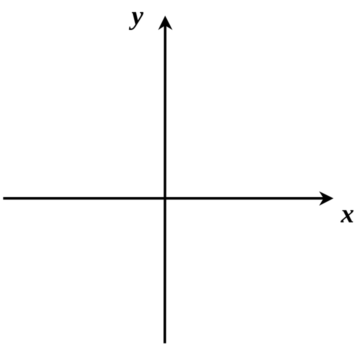
c On this set of axes sketch the general shape of the graph $y = kx^n$ when n is an odd integer greater than 1.
Describe what happens to the curve as n increases.



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2 a *Keeping n = 2, increase the value of k starting from 1. Change n to 3 and again increase k from 1.*
Describe what happens to the curves as k increases.

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b *Keeping n = 2, change k from – 1 to 1. Check if the same thing happens with other values of n and k when you change k from positive to negative.*

Describe what happens to the curve when you change k from positive to negative.

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3 *Keeping k = 1, start with n = 0.5 then click the scroll bar to the right of the divider to increase n to 1.5 then 2.5,* 3.5…. Why is there no curve when $x < 0$?

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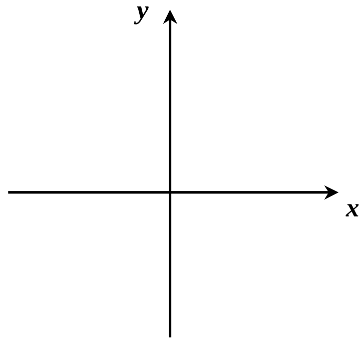
Use the fourth worksheet (y = kx^-n) of Interactive Graphs.xls

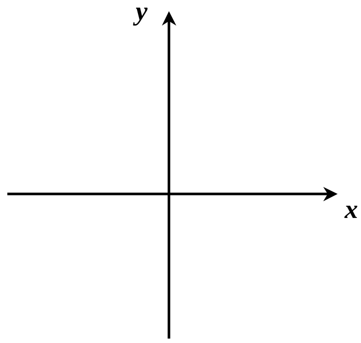
It is set up to show graphs with equations of the form $y = kx^{-n}$ i.e for negative powers of *x*.

4 ***Keeping k = 1 click on the scroll bar for n to the right of the central divider so that the power of x changes from 0 to – 1, then – 2, – 3, ….– 10.***

a On this set of axes sketch the general shape of the graph $y = kx^{-n}$ when the power is a negative odd integer.



b



On this set of axes sketch the general shape of the graph $y = kx^{-n}$ when the power is a negative even integer.

5 a *Keep the power as – 2 (i.e. keep n = 2). Increase the value of k starting from 1.*
Change the power to – 3 and again increase k from 1.
Describe what happens to the curves as *k* increases.

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b *Keeping n the same (any value), change k from – 1 to 1. Check if the same thing happens with other values of n and k when you change k from positive to negative.*

Describe what happens to the curve when you change *k* from positive to negative.

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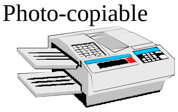


Teacher Notes

Unit Advanced Level, Working with algebraic and graphical techniques

Skills used in this activity:
- using Excel to investigate the shape of graphs

Preparation
Each student will need to have a copy of pages 1 to 6 and access to Excel. Change the name of the Excel file to 'Interactive Graphs.xls' and protect it if you wish before allowing students to use it.

Notes on Activity
This activity can be used to introduce the shape and main features of linear, quadratic and power graphs. It can be split into three separate activities to be done at different points in the course if you wish. (Note that an alternative version of the first part of the activity dealing with proportional and linear graphs can be found in Linear Graphs. This resource can be found in the skills activities section for Making connections in mathematics on the Nuffield website.)

Students should be told to always use the scroll bars to change the values of the constants.
Note that on the linear and quadratic worksheets, clicking on an arrow at the end of a scroll bar increases or reduces the constant by 0.1 whilst clicking on other parts of the bar increases or reduces the constant by 1.
On the power graph worksheets clicking on an arrow at the end of the scroll bar increases or reduces the constant by 0.5 whilst clicking on other parts of the bar increases or reduces the constant by 1.
Alternatively a constant can be varied by dragging the central bar along the scroll bar.

If you have the equipment needed to project the spreadsheet onto a screen, this will aid class discussion.
Discussion could include patterns in the values of x and y in the tables as well as features of the graphs.

Answers/Points for discussion

Linear Graphs

1 Lines with equations of the form $y = mx$ always pass through (0, 0) i.e. the origin.

2 As m increases from 0 to 5, the gradient of the line increases i.e. the line becomes steeper.

3 The gradient now becomes negative, with the line getting steeper as m decreases from 0 to – 5.

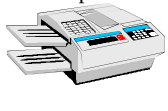
4 Lines with equations of the form $y = c$ have zero gradient.
As c increases from – 4 to 4 the line moves up the page from y = – 4 to y = 4.

5 For lines with equations of the form $y = x + c$**,** as c increases from – 4 to 4 the point of intersection of the line with the y axis moves from y = – 4 to y = 4 (and the gradient is 1).

6 For lines with equations of the form $y = -x + c$**,** as c increases from – 4 to 4 the point of intersection of the line with the y axis moves from y = – 4 to y = 4 (and the gradient is – 1).

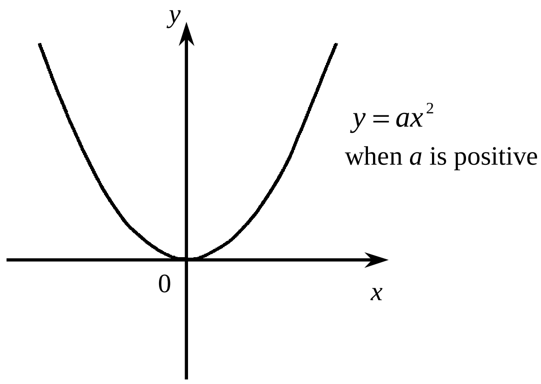
7 For all lines with equations of the form $y = mx + c$, c gives the intercept on the y axis.

8 For all lines with equations of the form $y = mx + c$, m gives the gradient.



Quadratic Graphs

1



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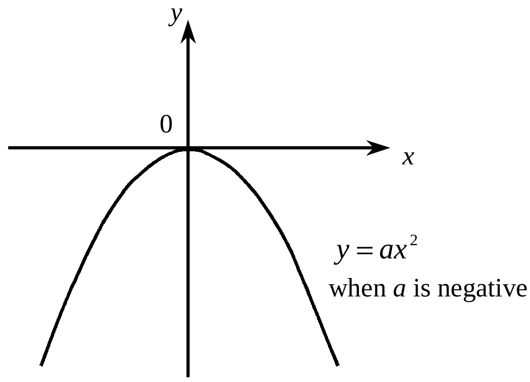


2 As a increases from 0 to 5 the curve ‘narrows’ and gets steeper.
4 As a decreases from 0 to – 5 the curve ‘narrows’ and gets steeper.
5 As c increases from – 4 to 4 the point of intersection of the curve with the y axis moves from – 4 to 4.
6 As c increases from – 4 to 4 the point of intersection of the curve with the y axis moves from – 4 to 4.
7 For quadratic curves with equations of the form $y = ax^2 + c$, c gives the intercept on the y axis.
The value of a affects the gradient and orientation of the curve.
8 As b increases from 0 to 4 the curve moves to the left and downwards.

In fact quadratic curves with equations of the form $y = x^2 + bx$ always cross the x axis at the points (0, 0) and (– b, 0). The minimum point is $\left(-\frac{b}{2}, -\frac{b^2}{4}\right)$.

9 As b decreases from 0 to – 4 the curve moves to the right and downwards.
10 As b increases from 0 to 4 the curve moves to the right and upwards.

Quadratic curves with equations of the form $y = -x^2 + bx$ always cross the x axis at the points (0, 0) and (b, 0). The maximum point is $\left(\frac{b}{2}, \frac{b^2}{4}\right)$.

11 As b decreases from 0 to – 4 the curve moves to the left and upwards.

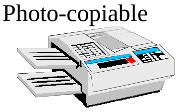
Summary for quadratic curves

Findings could include:

- The intercept on the y axis is always equal to c.
- In general changing either a or b changes the gradient as well as the position of the turning point of the curve.
- In the special case when a = 0 the graph is a straight line (unless b and c are also 0).
- When a is positive, increasing b always moves the turning point of the curve to the left.
When a is negative, increasing b always moves the turning point of the curve to the right.

Notes The turning point of a quadratic curve is given by $\left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right)$

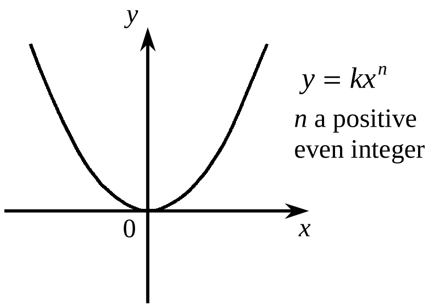
The gradient of a quadratic curve is given by $2ax + b$



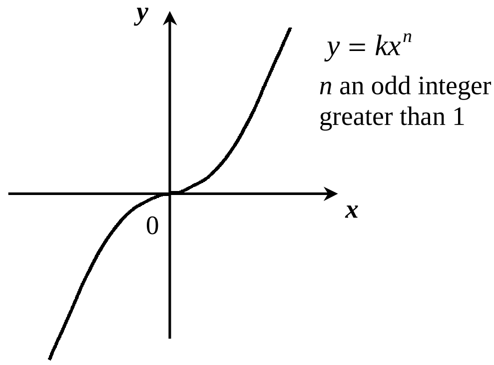
Power Graphs

1 a When $n = 0$ the equation reduces to $y = k$ and when $n = 1$ it is equivalent to $y = kx$.

b



c

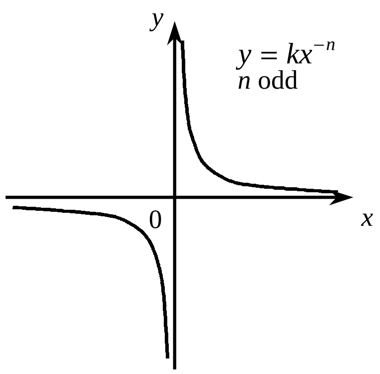


In both cases as n increases the curve becomes flatter for $-1 < x < 1$ and steeper for $x < -1$ and $x > 1$

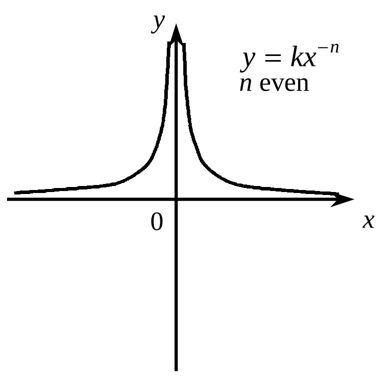
2 a As k increases the curve becomes steeper.
(The y value of each point on the curve $y = kx^n$ is equal to the y value on $y = x^n$ multiplied by k.)
b When k changes from positive to negative the curve is reflected in the x axis.

3 When n is 0.5, 1.5, 2.5,…the equations involve the square root of x.

4 a



b



5 a The y value of each point on the curve $y = kx^{-n}$ is equal to the y value on $y = x^{-n}$ multiplied by k.
b When k changes from positive to negative the curve is reflected in the x axis.